

**Dear Family,**

The next unit in your child's course of study is *Decimal Ops: Computing with Decimals and Percents*. It is the third and final unit in the development of understanding fractions, decimals, and percents. This unit will develop understanding and facility with decimal operations and percents.

**UNIT GOALS**

Like the work done to develop fraction operations, students will engage in many problem situations as they develop algorithms for adding, subtracting, multiplying, and dividing decimals. They will explore percents in the context of tip, tax, discount, and total cost.

Students have two ways of making sense of what decimals mean— extending the place value system on which our number system is built or interpreting decimals as fractions. These two ideas are related, but they have a different look and feel to students. In order to have the most complete understanding of and skill with computation, students need to understand each of these meanings. Then they use them to examine why decimal algorithms for addition, subtraction, multiplication, and division make sense. Depending on the operation, the fraction interpretation or the place value interpretation may help in finding short cut algorithms. Students will draw upon and use the ideas developed in *Comparing Bits and Pieces* and *Let's Be Rational*. For example, students will use the algorithm they developed for multiplying fractions in *Let's Be Rational* to help them develop and understand an algorithm for multiplying decimals.

**HELPING WITH HOMEWORK**

You can help with homework and encourage sound mathematical habits as your child studies this unit by asking questions such as:

- *Which operations on decimals or percents will help in solving this problem?*
- *What algorithms will help with the calculations?*
- *About how large will the sum, difference, product, or quotient be?*
- *What number is a reasonable solution to the problem?*
- *What do the decimals and/or percents involved tell me about the problem situation?*

In your child's notebook, you can find worked-out examples from problems done in class, notes on the mathematics of the unit, and descriptions of the vocabulary words.

**HAVING CONVERSATIONS ABOUT THE MATHEMATICS IN *DECIMAL OPS***

You can help your child with his or her work for this unit in several ways:

- Ask your child for an explanation of the ideas in a problem. For example, why do you line up the decimals when adding and subtracting decimal numbers?
- At times, students may be working with ideas and algorithms that are different from the ones you learned for adding, subtracting, multiplying, and dividing decimals. Encourage your child to share these methods with you as a way to help them make sense of what they are studying.
- When shopping or eating in a restaurant with your child, ask him or her to estimate what the tax will be on a purchase or what the tip should be for a meal.

A few important mathematical ideas that your child will learn in *Decimal Ops* are given on the back.

As always, if you have any questions or concerns about this unit or your child's progress in class, please feel free to call. We are all interested in your child and want to be sure that this year's mathematics experiences are enjoyable and promote a firm understanding of mathematics.

Sincerely, Mr. Hogan

Important Concepts	Examples
<p><b>Deciding on an Operation</b> Students look at various situations involving decimals and determine what operation is needed to find a solution. Students find a nearby “nice” whole numbers to the decimal numbers to decide what operations make sense.</p>	<p>Nora made a tablecloth in the shape of a rectangle that was 3.5 meters long and 1.5 meters wide. What is the area of the tablecloth? The tablecloth is about 4 meters long by 2 meters wide. If we want to find the area of a 4 by 2 rectangular tablecloth, we would multiply <math>4 \times 2</math> to get an answer of 8. Therefore, to find the exact answer, we would multiply <math>3.5 \times 1.5</math>.</p>
<p><b>Addition and Subtraction of Decimals</b> <u>Place Value Interpretation</u> Students use a clerk’s error to raise the issue of what the digits mean in a number and what that means for adding or subtracting numbers. <u>Decimals As Fractions</u> Write the decimals as fractions, find common denominators, add or subtract the fractions, and express the answers as decimals. This confirms that when adding or subtracting, one must compute with digits of the same value.</p>	<p>Zeke buys cider for \$1.97 and donuts for \$0.89. The clerk said the bill was \$10.87. What did the clerk do wrong?</p> <p>The cider is <math>\\$1.97 = \frac{197}{100}</math> and the donuts are <math>\\$0.89 = \frac{89}{100}</math>. So the cost is <math>\frac{89}{100} + \frac{197}{100} = \frac{286}{100} = 2.86</math>. This is like thinking of the cost in pennies and then writing the sum in dollars.</p> <p>In <math>3.725 - 0.41 = 3.315</math>, we subtract thousandths from thousandths (0.005 – 0.000), hundredths from hundredths (0.02 – 0.01), tenths from tenths (0.7 – 0.4), and ones from ones (3 – 0).</p>
<p><b>Multiplication of Decimals</b> <u>Decimals As Fractions</u> Students express decimals as fractions, multiply the fractions, write the answer as a decimal, and relate the number of decimal places in the factors to the answer. <u>Place Value Interpretation</u> Students look at sets of problems to see why counting decimal points makes sense. Finally, students use the short-cut algorithm: multiply the decimals as whole numbers and adjust the place of the decimal in the product.</p>	<p>We can look at a problem using equivalent fractions.</p> $0.3 \times 2.3 = \frac{3}{10} \times 2\frac{3}{10} = \frac{3}{10} \times \frac{23}{10}$ <p>The product as a fraction is <math>\frac{69}{100}</math>, as a decimal 0.69. The 100 in the denominator shows that there should be two decimal places (hundredths) in the answer. The denominator of the fraction tells us the place value needed in the decimal.</p> <p>Using the fact that <math>25 \times 31 = 775</math> students reason about related products: <math>2.5 \times 3.1</math> (tenths <math>\times</math> tenths or hundredths in the product) = 7.75; <math>2.5 \times 0.31</math> (tenths <math>\times</math> hundredths or thousandths in the product) = 0.775; <math>0.25 \times 0.31</math> (hundredths <math>\times</math> hundredths or ten thousandths in the product) = 0.0775.</p>
<p><b>Division of Decimals</b> <u>Decimals As Fractions</u> Students express decimals as fractions, get common denominators, and divide the numerators. <u>Place Value Interpretation</u> Write an equivalent problem by multiplying both the dividend and the divisor by the same power of ten until both are whole numbers.</p>	<p><math>3.25 \div 0.5 = \frac{325}{100} \div \frac{5}{10} = \frac{325}{100} \div \frac{50}{100}</math>. This is the same as <math>325</math> hundredths <math>\div 50</math> hundredths = <math>6\frac{1}{2}</math> or 6.5.</p> <p><math>37.5 \div 0.015 = \frac{375}{10} \div \frac{15}{1000} = \frac{37500}{1000} \div \frac{15}{1000} = 37,500 \div 15 = 2,500</math>. This makes a whole number problem with the same quotient as the original decimal problem.</p> <p>The fraction approach explains why moving decimal places works.</p> $0.015 \overline{)37.5} = 0.015 \times 1,000 \overline{)37.5 \times 1,000} = 15 \overline{)37500}$
<p><b>Decimal Forms of Rational Numbers</b> <u>Finite (or Terminating) Decimals</u>: Decimal forms for rational numbers that “end”. The simplified fraction form has only 2s or 5s in the prime factorization of the denominator.</p>	<p><math>\frac{1}{2} = 0.5</math>, <math>\frac{3}{4} = 0.75</math>, <math>\frac{1}{8} = 0.125</math>, <math>\frac{12}{75} = 0.16</math>.</p> <p>In simplified fraction form <math>\frac{12}{75} = \frac{4}{25}</math> has only factors of five (<math>4/5 \times 5</math>) in the denominator. A power of ten in the denominator makes the decimal terminate. <math>\frac{4}{25} = \frac{16}{100} = 0.16</math></p>
<p><u>Infinite Repeating Decimals</u>: Decimals that “go on forever” but show a repeating pattern. These are fractions with prime factors other than 2 or 5 in the simplest denominator form.</p>	<p><math>\frac{1}{3} = 0.333333\dots</math>, <math>\frac{2}{3} = 0.666666\dots</math>, <math>\frac{8}{15} = 0.5333333\dots</math>, <math>\frac{3}{7} = 0.42857142857142\dots</math></p> <p>In simplified fraction form <math>\frac{26}{150} = \frac{13}{75} = \frac{13}{(3 \times 5 \times 5)} = 0.1733333\dots</math></p>
<p><b>Finding Percents</b> <u>Percent of a Price</u> “Jill buys a CD for \$7.50. The sales tax is 6%. How much is the tax?”</p>	<p>6% of \$7.50 = cost of tax 1% of \$7.50 = <math>\frac{1}{100}</math> of \$7.50 = <math>\\$7.50 \div 100 = 0.075</math> 6 of the 1%’s will give me 6%. So, 6% of \$7.50 = \$0.45</p>
<p><u>On What Amount was the Percent Figured</u> “Customers left Jill \$2.50 as a tip. The tip was 20% of the total. How much was the bill?”</p>	<p>20% of some number equals \$2.50 One way is to ask how many 20%’s it takes to make 100%? In this case we need five. So, <math>5 \times \\$2.50</math> gives us \$12.50.</p>
<p><u>What Percent One Number is of Another Number</u> “Sam got a \$12 discount off a \$48 purchase. What percent discount did he get?” “80 out of 200 cat owners say their cat has bad breath. What percent is this?”</p>	<p>We have to find what % 12 is of 48. Students can informally solve this by computing how many 12s in 48. It takes four, so the percent is <math>\frac{1}{4}</math> of 100% or 25%. The same sort of reasoning is appropriate — what % of 200 equals 80. The fraction <math>\frac{80}{200}</math> as a decimal is 0.4 or 40%.</p>